exponents (n_δ) in [2] was taken from experiment, which permitted avoidance of additional hypotheses about the constance of ν_T or L across the wake.

NOTATION

U_i, u_i, mean and fluctuating velocity components; U_∞, free stream velocity; $\Delta U = U_1 - U_\infty$, velocity defect; $\overline{u_i u_j}$, Reynolds stress; $q^2 = \overline{u_i^2}$, twice the kinetic energy of turbulence; $\varepsilon = \nu\overline{(\partial u_i/\partial x_k)^2}$, rate of turbulene energy dissipation; d, maximal vertical body dimension; x_i, Cartesian coordinates; x_1, coordinate along the main flow; x_2, vertical coordinate; λ, Taylor microscale; $\eta_u = x_2/\delta_u$; $\eta_E = x_2/\delta_E$, dimensionless coordinates. Subscript: m, maximal value; 0, value on the axis; ∞, value in the free stream; $\overline{(\ )}$, mean value.

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MOTION OF VISCOELASTIC LIQUIDS IN A POROUS MEDIUM

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Nonlinear effects occurring in filtration of viscoelastic liquids are considered. The qualitative differences between one-dimensional and planar cases and between motion in homogeneous and inhomogeneous porous media are demonstrated.

Motion of viscoelastic liquids in a porous medium can be described by a filtration law of the form [1-3]

$$L_1\mathbf{u} = -\frac{k}{\mu}\,\mathrm{grad}\,(L_2 P), \tag{1}$$

where the operators L_1, L_2 have the form

$$L_i = 1 + \sum_{k=1}^{n_i} T_k^i \partial_t^k,$$

where T_k^i are the relaxation time spectra.

These models permit consideration of the unique features of viscoelastic liquid filtration, the major one of which is lengthening of transient processes in the porous medium,

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caused by the relaxation time spectrum. We will note that Eq. (1) is linear. Numerous experimental studies indicate that relaxation times are of the order of magnitude of 10^3-10^5 sec.

We will consider a generalization of the model of Eq. (1), using the full time derivatives in writing the filtration law. Within the framework of this model one can obtain a description of the nontrivial qualitatively unique features of viscoelastic liquid filtration observed in corresponding experiments. It should be stressed that these effects are produced by the nonlinear character of the equations obtained, while use of partial time derivatives in the filtration law leads only to an extension of transient processes.

It should also be noted that nonlinear effects are also observed in a steady-state viscoelastic liquid filtration regime [4].

Commencing from these facts, we will write the filtration law in the form

$$\mathbf{u} = -\frac{k}{\mu}\left[\nabla P + T\left(\frac{\partial}{\partial t}\nabla P + \frac{1}{m}(\mathbf{u}\nabla)\nabla P\right)\right]. \tag{2}$$

Thus, Eq. (2) differs from Eq. (1) in that it contains the full time derivative. For viscous liquids the corresponding nonlinear terms are small and can be neglected. Under the conditions considered herein, the coefficient standing before these terms, the relaxation time T, is quite large and these terms cannot be dropped.

We note that at low filtration velocities where $uT \ll lm$, the nonlinear term in Eq. (2) can be neglected, whereupon the expression reduces to filtration law (1). We can introduce the Debour number $D = uT(lm)^{-1}$ such that the nonlinear terms are insignificant at $D \ll 1$.

Traditionally, the filtration properties of a petroleum stratum have been studied in a core sample with steady-state regime. Under such conditions the velocity and pressure gradient are constant, and we can obtain Darcy's law from Eq. (2). At the same time, the situation changes for steady-state, but non-one-dimensional flows. For example, we will consider planar filtration. We have:

$$u_x = -\frac{k}{\mu}\left[\frac{\partial P}{\partial x} + \frac{T}{m}\left(u_x\frac{\partial^2 P}{\partial x^2} + u_y\frac{\partial^2 P}{\partial x \partial y}\right)\right],$$

$$u_y = -\frac{k}{\mu}\left[\frac{\partial P}{\partial y} + \frac{T}{m}\left(u_x\frac{\partial^2 P}{\partial x \partial y} + u_y\frac{\partial^2 P}{\partial y^2}\right)\right],$$

whence we obtain

$$u_x = -\frac{k}{\mu}\frac{\left(1 + \frac{Tk}{m\mu}\frac{\partial^2 P}{\partial y^2}\right)\frac{\partial P}{\partial x} - \frac{Tk}{m\mu}\frac{\partial^2 P}{\partial x \partial y}\frac{\partial P}{\partial y}}{\left(1 + \frac{Tk}{m\mu}\frac{\partial^2 P}{\partial x^2}\right)\left(1 + \frac{Tk}{m\mu}\frac{\partial^2 P}{\partial y^2}\right) - \frac{kT^2}{\mu^2 m^2}\left(\frac{\partial^2 P}{\partial x \partial y}\right)^2},$$

$$u_y = -\frac{k}{\mu}\frac{\frac{Tk}{\mu m}\frac{\partial^2 P}{\partial x \partial y}\frac{\partial P}{\partial x} - \left(1 + \frac{Tk}{\mu m}\frac{\partial^2 P}{\partial x}\right)\frac{\partial P}{\partial y}}{\left(1 + \frac{Tk}{\mu m}\frac{\partial^2 P}{\partial x^2}\right)\left(1 + \frac{Tk}{\mu m}\frac{\partial^2 P}{\partial y^2}\right) - \frac{k^2 T}{\mu^2 m^2}\left(\frac{\partial^2 P}{\partial x \partial y}\right)^2}.$$

Thus, the filtration equations are nonlinear, so that steady-state flows will differ qualitatively in the linear and spatial cases.

We will analyze liquid motion in an inhomogeneous medium. Let the relationship k(x) for one-dimensional flow be known. Then Eq. (2) gives

$$u = -\frac{k(x)}{\mu}\left(\frac{dP}{dx} + \frac{Tu}{m}\frac{d^2 P}{dx^2}\right) = \text{const.} \tag{3}$$

In formulating the problem we require not two, as is usually the case, but three boundary conditions. We have the two traditional conditions: $P(0) = P_0$; $P(l) = 0$. We write the third condition in the form: $\frac{d^2P(0)}{dx^2}=0$, or, considering Eq. (3), $u(0) = -\frac{k(0)}{\mu}\frac{dP(0)}{dx}$. Physically, this condition means that in the initial section whence the filtration flow commences, the relaxation properties do not manifest themselves and the filtration rate is determined by Darcy's law. For motion in a porous medium with constant permeability this condition ensures a linear pressure distribution over specimen length and a linear relationship between the pressure head and the flow rate, as has been observed in experiment.

We will first consider the case where the permeability decreases with length. For definiteness, we take $k(x) = k_0\exp(-bx)$.

Solving this problem, we find

$$P_0=\frac{\mu u^2T}{k_0 m}-\frac{\mu u}{bk_0}-\frac{\mu u}{bk_0\left(1+\frac{Tub}{m}\right)}\exp(bl)+\left(\frac{\mu u}{bk_0}-\frac{T\mu u^2}{mk_0}-\frac{\mu u}{bk_0\left(1+\frac{Tub}{m}\right)}\right)\exp\left(-\frac{lm}{Tu}\right). \quad (4)$$

For motion in the direction of increasing permeability we take $k_*(x)=k_0\exp[-b(l-x)]$. This case can be reduced to the one considered previously by taking $k_*(x)=k_1\exp(bx)$, where $k_1=k_0\exp(-bl)$. Then, replacing k_0 by $k_0\exp(-bl)$ in Eq. (4) and changing the sign of the parameter b, we obtain

$$P_{01}=\left(-\frac{\mu u^2T}{k_0}+\frac{\mu u}{bk_0}\right)\exp(bl)-\frac{\mu u}{bk_0\left(1-\frac{Tub}{m}\right)}+$$
$$+\left[\frac{\mu u}{bk_0\left(1-\frac{Tub}{m}\right)}-\frac{\mu u}{bk_0}-\frac{\mu u^2T}{k_0}\right]\exp\left(bl-\frac{lm}{Tu}\right).$$

Calculations with Eq. (4) show that depending on the law of permeability change over length the filtration rate will differ for an identical pressure head. Thus for motion in the direction of increasing permeability the filtration rate proves to be higher than for motion in the opposite direction.

Analysis reveals that for low values of relaxation time the difference between pressure heads for change in the direction of motion is insignificant, i.e., $D \ll 1$. In the other limiting case, where $D \gg 1$, the liquid does not succeed in relaxing, and the effect in question is also not observed. The major contribution to resistance is then given by the term $\sim T$. Under intermediate conditions pressure heads will differ by a factor of several times.

We will now consider planar radial filtration in a homogeneous stratum. In this case the function $P = P(r)$ is nonlinear and $d^2P/dr^2 \neq 0$. In connection with the fact that for filtration from the stratum into the well and from the well into the stratum d^2P/dr^2 changes sign, one should expect a change in filtration rates for one and the same pressure head. Below we will obtain an expression to estimate this "hysteresis" effect. In deriving the following expressions (as was done in deriving Eq. (2)) a positive velocity (flow rate) value corresponds to motion away from the origin of the coordinate system.

For filtration from the stratum into the well the boundary conditions are formulated as follows:

$$r=R_c \quad P=0;\ r=R \quad P=P_0,\ \frac{dP}{dr}=\frac{\mu Q}{2\pi khR}=\frac{\mu q}{kR}, \quad (5)$$

where $q = Q/2\pi h$; h being the thickness of the stratum.

Thus, the problem reduces to solution of the equation

$$\frac{d^2P}{dr^2}-r\frac{m}{Tq}\frac{dP}{dr}=-\frac{\mu m}{kT} \quad (6)$$

for the boundary conditions of Eq. (5).

We reduce Eq. (6) to dimensionless form:

$$\frac{\beta}{\alpha^2}\frac{d^2P}{dr_*^2} - \frac{\alpha r_* m\beta}{Tq\alpha}\frac{dP_*}{dr_*} = -\frac{\mu m}{kT}.$$

Taking $\alpha r_* = r;\ \beta P_* = P;\ \alpha = \sqrt{Tq/m};\ \beta = q\mu/k$, in place of the preceding expression, we obtain

$$\frac{d^2P}{dr^2} - r\frac{dP}{dr} = -1 \tag{7}$$

(we omit the subscript * in Eq. (7) and below).

In dimensionless form, boundary conditions Eq. (5) can be written as:

$$r = R_c\ \ P = 0;\ r = R\ \ P = P_0,\ \frac{dP}{dr} = \frac{1}{R}. \tag{8}$$

The integral of Eq. (7) with boundary conditions (8) has the form

$$P_0 = \frac{1}{R}\exp\left(-\frac{R^2}{2}\right)\int\limits_{R_c}^{R}\exp\left(\frac{r}{2}\right)^2 dr + \int\limits_{R_c}^{R}\exp\left(\frac{r^2}{2}\right)\int\limits_{r}^{R}\exp\left(-\frac{\eta^2}{2}\right)d\eta dr. \tag{9}$$

It follows from Eq. (9) that the dependence of flow rate on pressure head for a viscoelastic liquid is nonlinear.

For motion from the well into the stratum the following boundary conditions exist:

$$r = R_c\ \ P = P_0,\ \frac{dP}{dr} = -\frac{1}{R_c};\ r = R\ \ P = 0. \tag{10}$$

The integral of Eq. (7) with boundary conditions (10) has the form

$$P_0 = \frac{1}{R_c}\exp\left(\frac{R_c^2}{2}\right)\int\limits_{R_c}^{R}\exp\left(-\frac{r^2}{2}\right)dr + \int\limits_{R_c}^{r}\exp\left(-\frac{r^2}{2}\right)\int\limits_{R_c}^{R}\exp\left(\frac{\eta^2}{2}\right)d\eta dr. \tag{11}$$

Comparison of Eqs. (9) and (11) shows that the flow rate of a viscoelastic liquid during filtration with a constant pressure head depends on the direction of the process — Eqs. (9) and (11) are not identical.

Experiments were performed with a poured radial model of a stratum in the form of a circular sector with outer radius 0.55 m and linear radius of 0.01 m (well contour), aperture angle of 18° and height 0.05 m. In the experiments steady state filtration of a polymer solution was carried out for different flow directions ("from the well," "into the well") by plotting indicator lines.

The porous medium was formed by densely packed quartz sand with grain diameter ≤0.1 mm. An aqueous solution of polyacrylamide with concentration varying from 0 to 0.2% by weight was used as the viscoelastic liquid. Before performing each experiment at a given concentration level liquid was passed through the model until the liquid viscosity at the output equalled that at the input. In all cases the quantity of solution required did not exceed five times the pore volume of the model. After doing this a steady state liquid filtration regime was established, i.e., for constant flow rate pressure at the input and output of the model were stabilized and readings made.

The flow rate of the polymer solution was measured by the weight method over the course of 20 min. To eliminate incidental effects during filtration with a specified polyacrylamide concentration the flow direction at various pressure heads was varied randomly. Results of the study are shown in Fig. 1. As is evident from Fig. 2, change in the system filtration resistance coefficient, which can be characterized by the ratio of flow rates for extraction

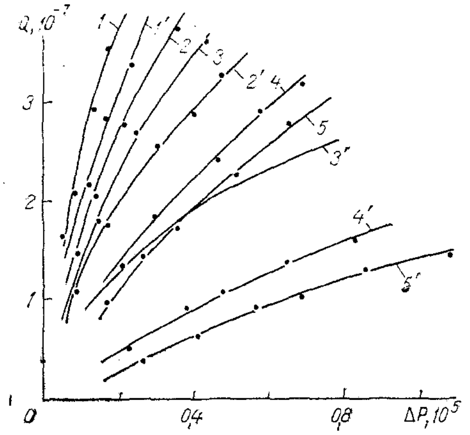

Fig. 1

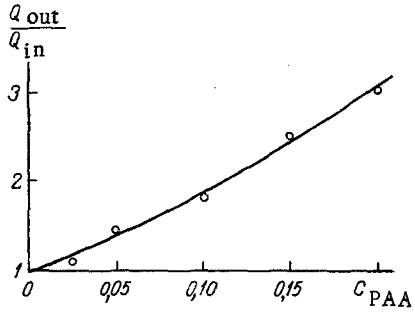

Fig. 2

Fig. 1. Flow rate of aqueous solution of polyacrylamide Q (m^3/sec) vs pressure head ΔP (Pa) for radial steady-state filtration: 1-5) flow into "well"; 1'-5') pumping into "stratum"; 1, 1') 0.025% PAA; 2, 2') 0.05; 3, 3') 0.10; 4, 4') 0.15; 5, 5') 0.020.

Fig. 2. Flow rate "hysteresis" Q_{out}/Q_{in} for polymer solution vs polyacrylamide concentration C_{PAA} (wt. %).

and pumping into the well for identical pressure heads, shows a tendency to increase with increase in the viscoelastic properties of the solution, since it is obvious that with increase in polyacrylamide concentration the rheological properties of the liquid change in proportion to this concentration.

NOTATION

P, pressure; u, filtration velocity; k, permeability; m, proosity; T, relaxation time; μ, viscosity; x, y, coordinates; t, time; l, length; D, Debour number, a dimensionless parameter.

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